



Student Name: _____

HW#1 CVL851: Special Topics in Transportation, Spring 2023

Problem 1 (10 Points) State Peano's axioms. (**Note:** Use wikipedia or any textbook, or any other resource for proper axiomatic definitions)

Problem 2 (10 Points)

1. Prove that the cardinality of the set of natural numbers \mathbb{N} , i.e. $\{0, 1, 2, \dots\}$ and the set of positive integers \mathbb{Z}^+ , i.e. $\{1, 2, \dots\}$ is the same.
 2. Give a set whose cardinality is higher than the cardinality of the set of real numbers \mathbb{R} .
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Problem 3 (10 Points) Prove that the set of rational numbers \mathbb{Q} is countable.

Problem 4 (10 Points) Prove that the interval $[0, 1] \in \mathbb{R}$ is uncountable.

Problem 5 (15 Points) Define a groupoid, a semigroup, a monoid, a group, a ring, a field, and a vector space. (**Note:** Use wikipedia or any textbook, or any other resource for proper axiomatic definitions).

Problem 6 (5 Points) State the fundamental theorem of algebra.

Problem 7 (10 Points) In a complex field what is the multiplicative inverse of $2 + i3$, and in quaternions field, what is the product of $(2 + 3i)(1 + i - 4j + 5k)$?

Problem 8 (10 Points) Prove that the set $\mathbb{Z}_3 = \{0, 1, 2\}$ with modulo 3 arithmetic is a ring. Modulo arithmetic is similar to *clock* arithmetic which is modulo 12 where 12 is same as 0, and therefore 10+3 which would be 13 is same as $13 - 12 = 1$.

Problem 9 (10 Points) Prove that the set polynomials with coefficients in \mathbb{R} is a vector space over the real field \mathbb{R} .

Problem 10 (10 Points) Draw a unit circle for \mathbb{R}^2 using ℓ_2 norm, ℓ_1 norm, and ℓ_∞ norm, on the same plot.

