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Problem 1 (25 Points) Given the following table for a groupoid $\{\Omega, \odot\}$.

$\odot$	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

1. Find the identity element of the groupoid.
2. Fill the following table

$(a \odot b) \odot c =$	☐	$a \odot (b \odot c) =$	☐
$(a \odot c) \odot b =$	☐	$a \odot (c \odot b) =$	☐
$(b \odot c) \odot a =$	☐	$b \odot (c \odot a) =$	☐
$(b \odot a) \odot c =$	☐	$b \odot (a \odot c) =$	☐
$(c \odot a) \odot b =$	☐	$c \odot (a \odot b) =$	☐
$(c \odot b) \odot a =$	☐	$c \odot (b \odot a) =$	☐

   Is this groupoid a semigroup?
3. Is this groupoid a monoid?
4. Fill the following table

$a \odot b =$	☐	$b \odot a =$	☐
$a \odot c =$	☐	$c \odot a =$	☐
$b \odot c =$	☐	$c \odot b =$	☐

   Is this groupoid abelian?
5. Find the inverses of a, b, and c. Is this groupoid a group?

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Problem 2 (20 Points)

1. Find the angle between the vectors $x_1 = [1\ 0\ -1\ 0]'$, and $x_1 = [-1\ 1\ 1\ -1]'$ using the ℓ_2 Hilbert space on $\mathbb{R}^4$.
2. Find the angle between the functions $f_1(\theta) = \sin\theta$, and $f_2(x) = \cos\theta$ using the L_2 Hilbert space defined on $\theta \in [0, \pi]$. Are they orthogonal?
3. Find the L_2 limit of the series of functions $f_n(x) = x^n$ as $n \to \infty$ defined on $[0, 1]$.
4. Find the norm $\|(x_n)\|$ of the sequence $x_n = 1/\sqrt{3^n}$, $n = 1, 2, \cdots$ in ℓ_2. Notice that this is an infinite sequence, and hence, just use infinite sum for the norm extending the idea of ℓ_2 from $\mathbb{R}^n$.

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Problem 3 (5 Points) The set $X_1 = \{a, b, c\}$ has the topology $\mathcal{T}_1 = \{\phi, \{a\}, \{a, b\}, X_1\}$. Similarly, the set $X_2 = \{0, 1\}$ has the topology $\mathcal{T}_2 = \{\phi, \{0\}, \{1\}, X_2\}$. Is the function $f : X_1 \to X_2$, where $f(a) = 0$ and $f(b) = f(c) = 1$ continuous? Show why or why not.

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Problem 4 (10 Points) The set $X = \{a, b, c\}$ has the topology $\mathcal{T} = \{\phi, \{a\}, \{b, c\}, X\}$. What is the limit of the following sequences? Give reason why for each.

1. $x = (a, a, b, b, b, b, b, b, \cdots, b, b, b, \cdots)$, that is, all bs after first two as.
2. $x = (b, b, a, a, a, a, a, a, \cdots, a, a, a, \cdots)$, that is, all as after first two bs.
3. $x = (a, b, a, b, a, b, \cdots)$, that is, alternate as and bs.