Problem 1 (10 Points) **Derive** the laplace transform of the following signals:

1. $x(t) = e^{-at}u(t),\ a$ real.
2. Dirac distribution $\delta(t)$
3. unit step function $u(t)$

Problem 2 (10 Points) Using tables or directly, find the Laplace transform of the following:

1. $x(t) = u(t - t_0)$
2. $x(t) = e^{-2t}[u(t) - u(t - 5)]$
3. $x(t) = \delta'(t)$

Problem 3 (10 Points) Find the inverse Laplace transform of:

$$X(s) = \frac{s^2 + 2s + 5}{(s + 3)(s + 5)^2}, \quad \text{Re}(s) > -3$$

Problem 4 (10 Points) Using Unilateral Laplace transform solve the following differential equation:

$$\frac{dx(t)}{dt} + ax(t) = 0, \quad x(0) = x_0$$

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$	$R' \supset R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	$R' = R$
Shifting in s	$e^{s_0t}x(t)$	$X(s - s_0)$	$R' = R + \text{Re}(s_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X(s)$	$R' = aR$
Time reversal	$x(-t)$	$X(-s)$	$R' = -R$
Differentiation in t	$\frac{dx(t)}{dt}$	$sX(s)$	$R' \supset R$
Differentiation in s	$-tx(t)$	$\frac{dX(s)}{ds}$	$R' = R$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$	$R' \supset R \cap \{\text{Re}(s) > 0\}$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$