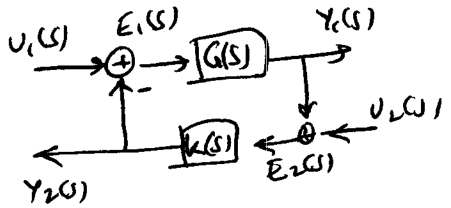
ECE 5404 HW#5 Solutions Spring 04

(5.1)



$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} (1+KG)^{-1} & -K/(1+KG) \\ G/(1+KG) & (1+GK)^{-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\therefore G_{e_1u_1}(s) = \frac{1}{1+K(s)G(s)} ; \quad G_{e_1u_2}(s) = -\frac{K(s)}{1+K(s)G(s)}$$

$$G_{e_2u_1}(s) = \frac{G(s)}{1+G(s)K(s)} ; \quad G_{e_2u_2}(s) = \frac{1}{1+G(s)K(s)}$$

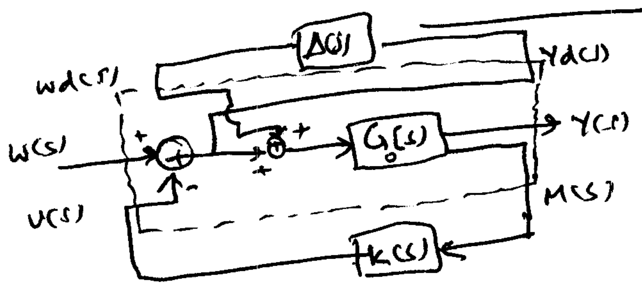
The pole of all four are the zeroes of $1+G(s)K(s)$.

$\therefore$ The SISO feedback system is internally stable iff all the zeroes of $1+G(s)K(s)$ have no positive real parts.

(5.3) MATLAB: num = [750000]; den = [1 350 15000 0]; [G P] = margin(num, den);

$GM^+ = 7.0$; $PM = 44.58°$

(5.4)



(5.6) $E_1(s) = G_{e_1u_2}(s)U_2(s) = (I - \Delta N_{y_dw_d})^{-1} U_1(s)$

$\Rightarrow E_1(s) = \Delta(s) N_{y_dw_d}(s) E_1(s) + U_1(s)$

$\therefore \|e_1(t)\|_2 \le \|\Delta N\|_\infty \|e_1(t)\|_2 + \|u_1(t)\|_2$

or $\|e_1(t)\|_2 \le (1 - \|\Delta\|_\infty \|N_{y_dw_d}\|_\infty)^{-1} \|u_1(t)\|_2$

$\therefore$ If $\|\Delta(s)\|_\infty \le 1$ and $\|N_{y_dw_d}\|_\infty < 1$

then $\dfrac{\|e_1(t)\|_2}{\|u_1(t)\|_2}$ gain is finite. $\therefore G_{e_1u_1}(s)$ is stable.

(5.7) We need to calculate $N_{y_dw_d}(s)$. We get different formula for $N_{y_dw_d}(s)$ depending on what type of uncertainty we pick from FIGURE 5.7. (a) $N_{y_dw_d}(s) = \dfrac{-K(s)}{1+K(s)G(s)}$; (b) $-\dfrac{GK}{1+GK}$; (c) $\dfrac{-GK}{1+GK}$

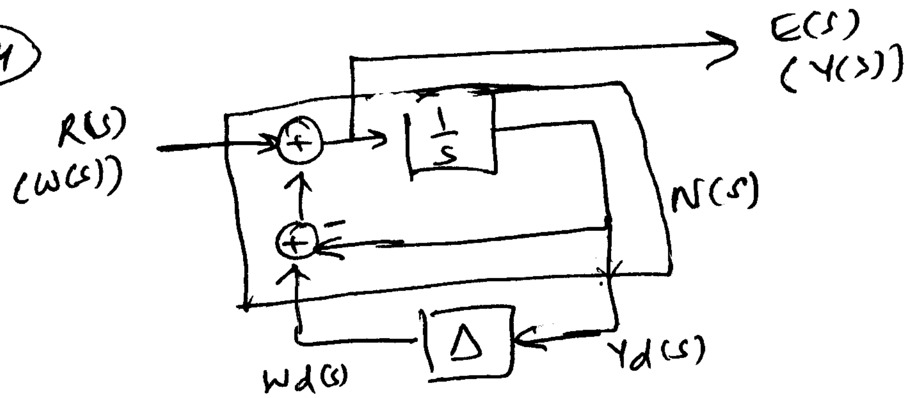
(d) $\dfrac{1}{1+GK}$, (e) $\dfrac{-1}{1+GK}$; For stability $\|N_{y_dw_d}\|_\infty$ should be less than 2.

As an example take (d) and (e), then using MATLAB "norm" function

$$\|N_{y_dw_d}(s)\|_\infty = \left\| \frac{1}{1+GK} \right\|_\infty = \left\| \frac{s^3+8s^2+4s+32}{s^3+8s^2+44s+112} \right\|_\infty = 1.7546 < 2$$

$\therefore$ robustly stable

5.14



(a) $$N_{y_d w_d}(s) = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

$$N_{y_d w}(s) = \frac{1}{s+1}$$

$$N_{y w_d}(s) = \frac{1}{1+1/s} = \frac{s}{s+1}$$

$$N_{yw}(s) = \frac{s}{s+1}$$

$$\therefore N = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} \\ \frac{s}{s+1} & \frac{s}{s+1} \end{bmatrix}$$

(b) $$\mu_{\underline{\Delta}}(N_{y_d w_d}) = \frac{1}{\min_{\Delta \in \underline{\Delta}} \left[\bar{\sigma}(\Delta) \text{ s.t. } \det(I + N_{y_d w_d} \Delta) = 0 \right.}$$

$$1 + N_{y_d w_d}(j\omega)\Delta(j\omega) = 0$$

$$\Rightarrow \quad \Delta(j\omega) = -(j\omega + 1)$$

$$\bar{\sigma}(\Delta) = \sqrt{1+\omega^2}$$

$$\therefore \text{SSV} = \frac{1}{\sqrt{1+\omega^2}}$$

(c) $$\sup_{\omega} \left[\frac{1}{\sqrt{1+\omega^2}} \right] = 1$$