

# ECE 5404: Multivariable Control

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Test#4

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Each problem is worth 10 points.

1. Find the minimizing control and the corresponding minimum cost for the system

$$\dot{x}_1(t) = 2x_1(t)$$

$$\dot{x}_2(t) = -3x_2(t) + u(t)$$

$$J(x(\cdot), u(\cdot)) = \int_0^{\infty} \{x^T(t) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + u^2(t)\} dt$$

2. Assuming that the assumptions of the Kalman Filter are true, is the following true? Prove your answer.

$$E[\{x(t) - \hat{x}(t)\}x^T(t)] = E[\{x(t) - \hat{x}(t)\} \{x(t) - \hat{x}(t)\}^T]$$

3. Design a feedback controller that minimizes

$$J(x(\cdot), u(\cdot)) = E[25x^2(\infty) + u^2(\infty)]$$

for the following system:

$$\dot{x}(t) = 5x(t) + 3u(t) + 2w(t)$$

$$m(t) = 3x(t)$$

Given that  $S_w = 1$

4. Design an LQR based control design such that the steady state value for the output  $y(t)$  is 2 for the following system.

$$\dot{x}(t) = 3x(t) + u(t)$$

$$y(t) = 5x(t)$$

You can take the value of any weights to be unity.