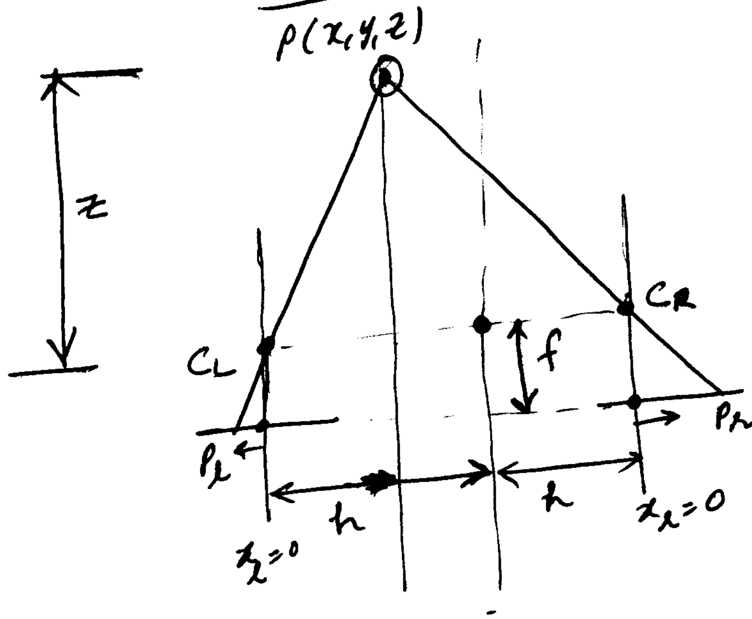
STEREO VISION / (MOTION DETECTION interpretation)



$$\frac{P_l}{f} = -\frac{h+x}{z} \quad -(1)$$

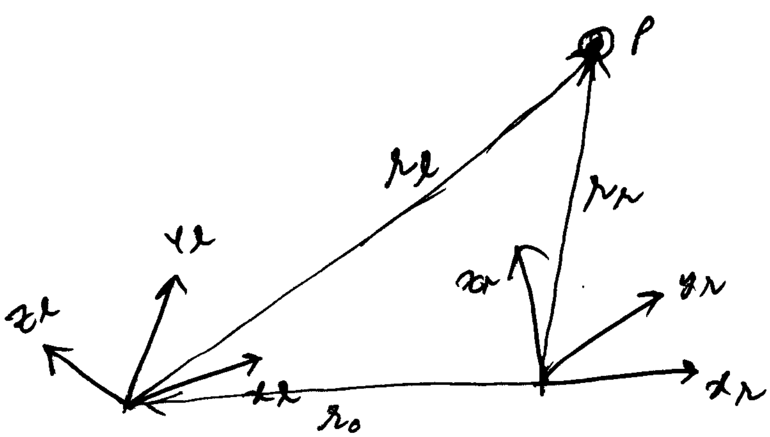
$$\frac{P_r}{f} = \frac{h-x}{z} \quad -(2)$$

$$(1) + (2) \Rightarrow z(P_r - P_l) = 2hf$$

$$\boxed{z = \frac{2hf}{P_r - P_l}}$$

ABSOLUTE ORIENTATION

$$r_r = R r_l + r_o$$



$$\boxed{\begin{aligned} r_{11} x_l + r_{12} y_l + r_{13} z_l + r_{14} &= x_r \\ r_{12} x_l + r_{22} y_l + r_{23} z_l + r_{24} &= y_r \\ r_{31} x_l + r_{32} y_l + r_{33} z_l + r_{34} &= z_r \end{aligned}}$$

Given (x_r, y_r, z_r) and (x_l, y_l, z_l) find R and r_o.

In general: use multiple points.
Define error for ith point $e_i = (R r_{l,i} + r_o) - r_{r,i}$
Least square solution parameters obtained by

$$\boxed{\operatorname{argmin} \sum_i |e_i|^2}$$

Relative Orientation

Projection of $r_l = (x_l, y_l, z_l)$ is

$$x_l' = \frac{x_l f}{z_l} \; ; \quad y_l' = \frac{y_l f}{z_l}$$

of $r_r = (x_r, y_r, z_r)$ is

$$x_r' = \frac{x_r f}{z_r} \; ; \quad y_r' = \frac{y_r f}{z_r}$$

We need to find R and r_0 given (x_l', y_l') and (x_r', y_r')

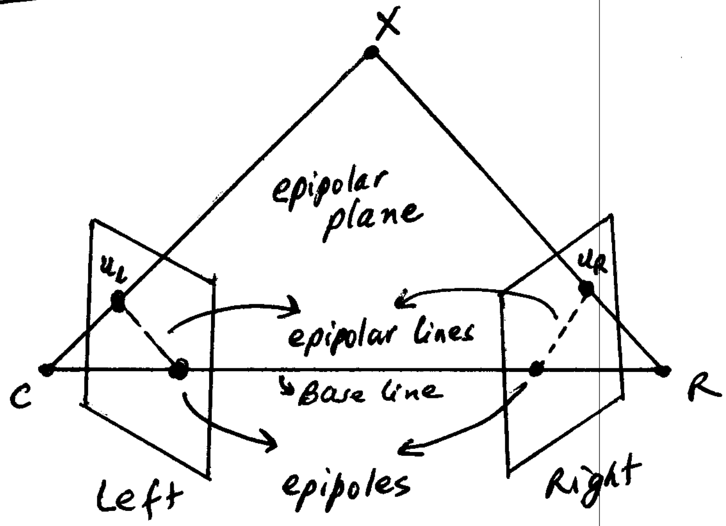
We have:

$$\boxed{\begin{aligned} r_{11} x_l' + r_{12} y_l' + r_{13} f + r_{14} \frac{f}{z_l} &= x_r' \frac{z_r}{z_l} \\ r_{21} x_l' + r_{22} y_l' + r_{23} f + r_{24} \frac{f}{z_l} &= y_r' \frac{z_r}{z_l} \\ r_{31} x_l' + r_{32} y_l' + r_{33} f + r_{34} \frac{f}{z_l} &= f \frac{z_r}{z_l} \end{aligned}}$$

- 3 equations & 14 unknowns $r_{11}, \dots, r_{34}, z_l, z_r$.
- Each additional point $\rightarrow$ 3 more equations; $\rightarrow$ 2 more unknowns, z_l, z_r
- $\frac{r_{14}}{z_l}, \frac{r_{24}}{z_l}, \frac{r_{34}}{z_l}, \frac{z_r}{z_l}$ are the variables. $\therefore$ If you have a solution for $r_{14}, r_{24}, r_{34}, z_r$ and z_l and you multiply all of those by a constant k, that is also a solution. $\therefore$ Solution is not unique. (The solution is <u>invariant</u> to scaling of all <u>distances</u>).
- To get a unique solution, we need some additional constraint, e.g. for $r_0 = (r_{14}, r_{24}, r_{34})^T$

$$\boxed{r_0 \cdot r_0 = 1}$$

EPIPOLAR GEOMETRY.



mathematical analysis

For left image

$$x_l = x_l' s, \quad y_l = y_l' s, \quad z_l = f s$$

In the right coordinate system

$$\left.\begin{aligned} x_r &= (r_{11} x_l' + r_{12} y_l' + r_{13} f) s + r_{14} \\ y_r &= (r_{21} x_l' + r_{22} y_l' + r_{23} f) s + r_{24} \\ z_r &= (r_{31} x_l' + r_{32} y_l' + r_{33} f) s + r_{34} \end{aligned}\right\} \quad (1)$$

they project to

$$x_r' = \frac{x_r f}{z_r}, \quad y_r' = \frac{y_r f}{z_r}$$

Rewrite (1) as

$$x_r = as + u, \quad y_r = bs + v, \quad z_r = cs + w$$

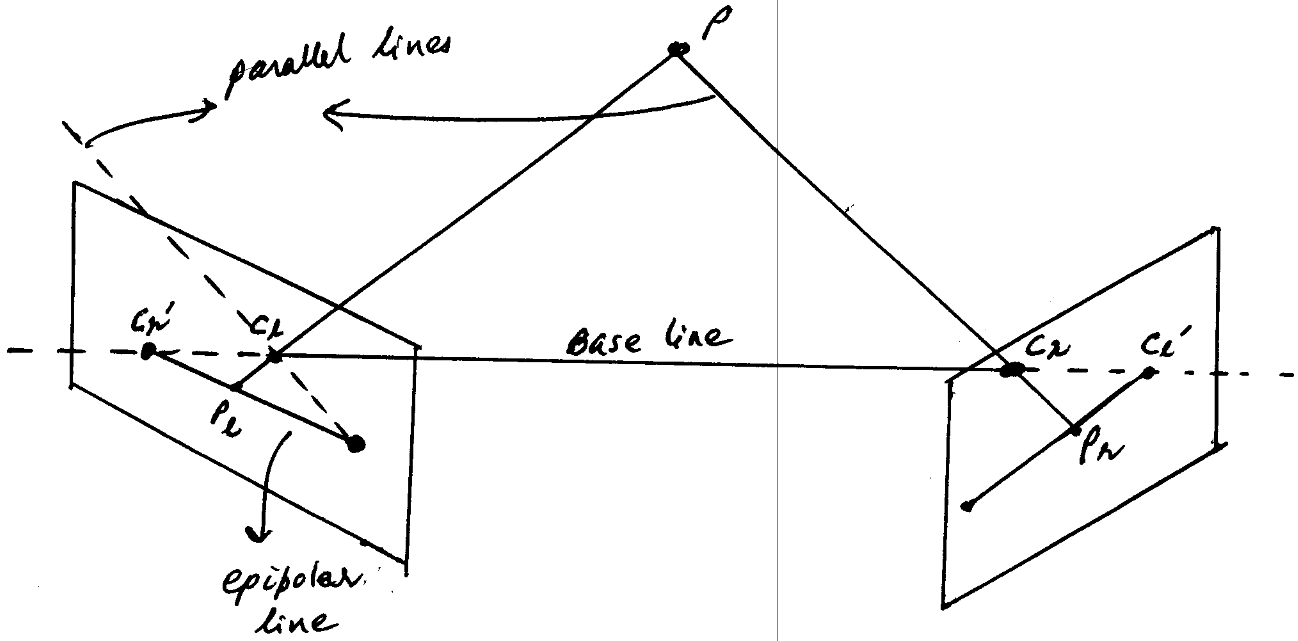
then

$$\left.\begin{aligned} \frac{x_r'}{f} &= \frac{a}{c} + \frac{cu - aw}{c} \cdot \frac{1}{cs + w} \\ \frac{y_r'}{f} &= \frac{b}{c} + \frac{cv - bw}{c} \cdot \frac{1}{cs + w} \end{aligned}\right] \quad (2)$$

For $s = 0$ gives $\frac{x_r'}{f} = \frac{u}{w}, \; \frac{y_r'}{f} = \frac{v}{w}$

$s \to \infty$ gives $\frac{x_r'}{f} = \frac{a}{c}; \; \frac{y_r'}{f} = \frac{b}{c}$

$\therefore$ (2) describes a straight line equation (epipolar line)



COMPUTING DEPTH

known R and r_0 between the two cameras.
Given (x_l', y_l') and (x_r', y_r') for same point on an object.

Equation

$$\left(r_{11}\frac{x_l'}{f} + r_{12}\frac{y_l'}{f} + r_{13}\right) Z_l + r_{14} = \frac{x_r'}{f} Z_r$$

$$\left(r_{21}\frac{x_l'}{f} + r_{22}\frac{y_l'}{f} + r_{23}\right) Z_l + r_{24} = \frac{y_r'}{f} Z_r$$

$$\left(r_{31}\frac{x_l'}{f} + r_{32}\frac{y_l'}{f} + r_{33}\right) Z_l + r_{34} = Z_r$$

use any two equations to compute Z_l and Z_r, and then

$$r_l = (x_l, y_l, z_l)^T = \left(\frac{x_l'}{f}, \frac{y_l'}{f}, 1\right)^T Z_l$$

$$r_r = (x_r, y_r, z_r)^T = \left(\frac{x_r'}{f}, \frac{y_r'}{f}, 1\right)^T Z_r$$

Exterior Orientation

subscript w for world coordinates
" c " camera "

$$\left.\begin{aligned} r_{11} x_a + r_{12} y_a + r_{13} z_a + r_{14} &= x_c \\ r_{21} x_a + r_{22} y_a + r_{23} z_a + r_{24} &= y_c \\ r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34} &= z_c \end{aligned}\right\} \quad \underline{E-1}$$

From images, $\frac{x'}{f} = \frac{x_c}{z_c}$; $\frac{y'}{f} = \frac{y_c}{z_c}$

$$\therefore \left.\begin{aligned} \frac{x'}{f} &= \frac{r_{11} x_a + r_{12} y_a + r_{13} z_a + r_{14}}{r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34}} \\ \frac{y'}{f} &= \frac{r_{21} x_a + r_{22} y_a + r_{23} z_a + r_{24}}{r_{31} x_a + r_{32} y_a + r_{33} z_a + r_{34}} \end{aligned}\right\} \quad \underline{E-2}$$

using least squares and given (x', y'), (x_a, y_a, z_a) for multiple points extract parameters.

Interior Orientation

from camera to image (including all affine effects)

$$\left.\begin{aligned} x' &= a_{11}(x_c/z_c) + a_{12}(y_c/z_c) + a_{13} \\ y' &= a_{21}(x_c/z_c) + a_{22}(y_c/z_c) + a_{23} \end{aligned}\right\} \quad \underline{I-1}$$

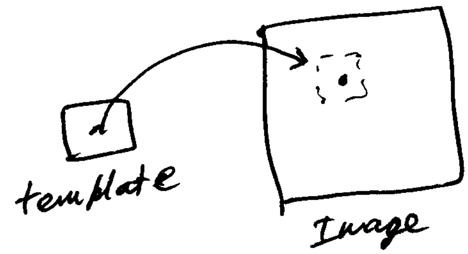
Combine I-1 with E-1 to get

$$\left.\begin{aligned} \frac{x'}{f} &= \frac{s_{11} x_a + s_{12} y_a + s_{13} z_a + s_{14}}{s_{31} x_a + s_{32} y_a + s_{33} z_a + s_{34}} \\ \frac{y'}{f} &= \frac{s_{21} x_a + s_{22} y_a + s_{23} z_a + s_{24}}{s_{31} x_a + s_{32} y_a + s_{33} z_a + s_{34}} \end{aligned}\right\} \quad \underline{I-2}$$

Finding Conjugate Points (The Correspondence Problem)

- Analyse images and identify features separately

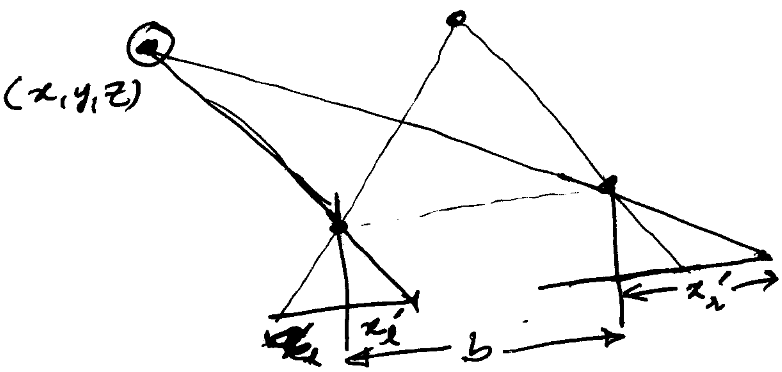
e.g. template matching



find the location of maximum correlation.

- Use the epipolar constraint.

Gray-Level Matching



$$\frac{x'_l}{f} = \frac{x+b/2}{z} \; ; \; \frac{x'_r}{f} = \frac{x-b/2}{z}$$

$$\frac{y'_l}{f} = \frac{y'_r}{f} = \frac{y}{z}$$

$$\frac{x'_l - x'_r}{f} = \frac{b}{z}$$

$$\therefore x = b\,\frac{(x'_l + x'_r)/2}{x'_l - x'_r}, \; y = b\,\frac{(y'_l + y'_r)/2}{x'_l - x'_r} \; ; \; z = \frac{bf}{x'_l - x'_r}$$

We want to find $z(x',y')$ $\left(z = bf/x'_l - x'_r\right)$ so that

$$I_l(x'_l, y'_l) = I_r(x'_r, y'_r)$$

or $I_l\left(f\,\frac{x+b/2}{z(x,y)}, y'\right) = I_r\left(f\,\frac{x-b/2}{z(x,y)}, y'\right)$

using $\frac{x'}{f} = \frac{x}{z}$ and $d(x',y') = \frac{bf}{z}$

Find $d(x', y')$ s.t

$$I_l\left(x' + \tfrac{1}{2}d(x',y'), y'\right) = I_r\left(x' - \tfrac{1}{2}d(x',y'), y'\right)$$

We want z and hence d to vary smoothly.

Find d that minimize

$$\iint \underbrace{\left[(E_l - E_r)^2 + \lambda(\nabla^2 d)^2\right]}_{F} dx'dy'$$

Calculus of Variations Problem : Solution obtained by <u>Euler equation</u>.

$$\hookrightarrow \quad F_d - \frac{\partial}{\partial x'} F_{d'_x} - \frac{\partial}{\partial y'} F_{d'_y} = 0$$

$$\Rightarrow \quad \nabla^2(\nabla^2 d) = \lambda (E_l - E_r)\frac{1}{2}\left(\frac{\partial E_l}{\partial x'} + \frac{\partial E_r}{\partial x'}\right)$$

Iterative Numerical schemes are available for this.

- Edge based methods