Region properties

Region properties

- Many properties can be extracted from an image region
  - area
  - length of perimeter
  - orientation
  - etc.
- These properties can be used for many tasks
  - object recognition
  - "dimensioning" (measuring sizes of physical objects)
  - to assist in higher-level processing

Common features and properties of regions

For simplicity, assume that $I(r,c) = \begin{cases} 1 & \text{for the region of interest} \\ 0 & \text{otherwise} \end{cases}$

- **Area** (or size): $A = \sum_r \sum_c I(r,c)$

  (this is just the number of pixels in the region)

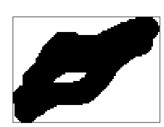
- **Width:** $w = 1 + \max_{I(r,c)=1}(c) - \min_{I(r,c)=1}(c)$

- **Height:** $h = 1 + \max_{I(r,c)=1}(r) - \min_{I(r,c)=1}(r)$

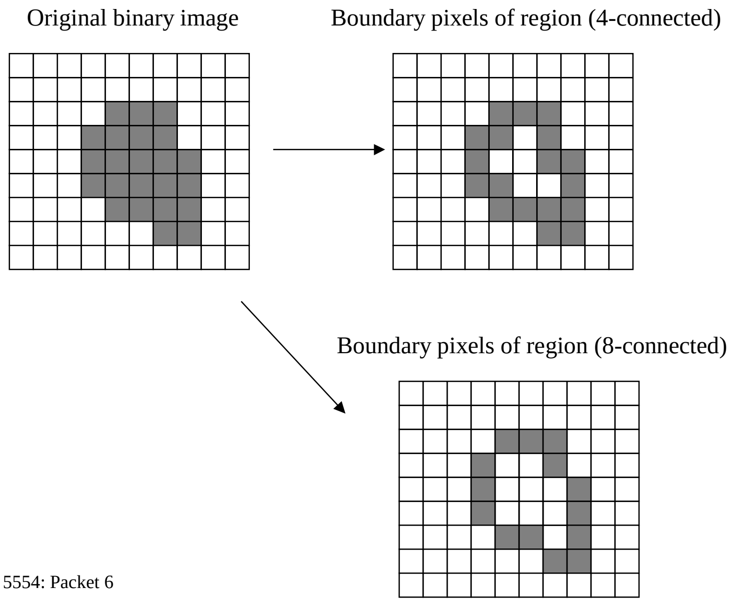
- **Bounding box** (= "bounding rectangle")

  Many properties that we compute for the region itself can be computed more quickly for the bounding box

  (And a rectangle is very easy to represent digitally; just store the coordinate locations of 2 opposite corners)



- **Region boundary** (= **border** or **perimeter**)



Let R represent the set of pixels (r, c) in a region

Assume that no holes are present in R

Let $N(r, c)$ represent the set of pixels that are neighbors of pixel (r, c)

The boundary of R is given by

$$B = \{(r,c) \mid (r,c) \in R \text{ and } N(r,c) - R \neq \emptyset\}$$

Notice: a 4-connected boundary results if 8-neigbors are considered, and vice versa

- **Length of region boundary**
  ("perimeter" sometimes refers to the length of the boundary)

The boundary is a closed digital curve

Represent the curve as a sequence of K pixels:

$$B' = \left((r_0, c_0), (r_1, c_1), \cdots, (r_{K-1}, c_{K-1})\right)$$

Let P represent the length of the boundary

There are 2 common ways to compute P:

P = number of pixels in the boundary = K

or

P = number of horizontal or vertical steps + $\sqrt{2} \times$ (number of diagonal steps)

- **Compactness** $= \dfrac{P^2}{A} \geq 4\pi$

  (This is minimum for a circular region.
  Sometimes compactness is called circularity.)

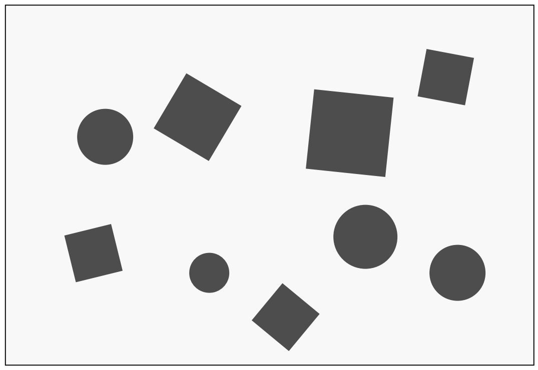
- **Elongation**:

  There is no single definition; commonly something of the form

  length / *width*

Example problem

Assume circles and squares are present in an image.
How can they be identified?



One possible approach

1. Threshold the image
2. Perform region labeling
3. For each separate region,
   - Compute compactness $\left(\frac{P^2}{A}\right)$
   - If compactness < 14 then
     classify the region as a circle
     else
     classify the region as a square

Circle: $\frac{P^2}{A} = 4\pi$

Square: $\frac{P^2}{A} = 16$

- **Centroid** (center of mass, center of gravity):

$$\begin{bmatrix} \bar{r} \\ \bar{c} \end{bmatrix} \quad \text{where} \quad \bar{r} = \frac{1}{A}\sum_r \sum_c r\, I(r,c)$$

$$\bar{c} = \frac{1}{A}\sum_r \sum_c c\, I(r,c)$$

- 2-D **moments**:

$$\boxed{m_{p,q} = \sum_r \sum_c r^p c^q I(r,c)}$$

Notice that $A = m_{00}$, $\bar{r} = m_{10} / m_{00}$, $\bar{c} = m_{01} / m_{00}$

- 2-D **central moments**:

$$\boxed{\mu_{p,q} = \sum_r \sum_c (r-\bar{r})^p (c-\bar{c})^q I(r,c)}$$

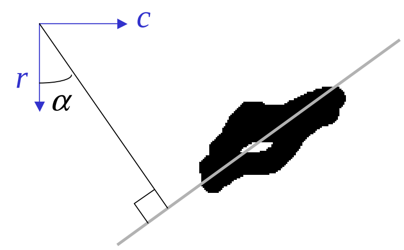
Notice that $\mu_{00} = A$

$\mu_{10} = 0 \qquad \mu_{01} = 0$

- 2nd-order moments can be used to determine orientation
- 2nd-order moments (and higher) are often used for recognition

- **Orientation**:

  Let α be the orientation relative to the *r*-axis



  It can be shown that

$$\tan 2\alpha = \frac{2\mu_{11}}{\mu_{20} - \mu_{02}}$$

- The central moments $\mu_{p,q}$ are *translation invariant*
- If we normalize by the area, we can also obtain *scale invariance*:

$$\boxed{\eta_{p,q} = \frac{\mu_{p,q}}{\mu_{0,0}^{\gamma}}}$$

where $\gamma = \frac{p+q}{2} + 1$

and $p + q = 2, 3...$

Moment invariants, Hu (1962)

- Hu introduced 7 functions of 2nd and 3rd moments that are invariant to translation, scale and rotation in 2 dimensions
- Here are the first 4:

$$\begin{aligned}
\phi_1 &= \eta_{20} + \eta_{02} \\
\phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\
\phi_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
\phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2
\end{aligned}$$

Affine moment invariants, Flusser and Suk (1993)

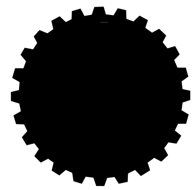
- Flusser and Suk derived functions of 2nd and 3rd moments that are invariant to any affine transformation in 2 dimensions
- Affine tranformation:

$$\hat{x} = a_1 x + a_2 y + d_1$$
$$\hat{y} = a_3 x + a_4 y + d_2$$

- Invariants:

$$I_1 = \frac{1}{\mu_{00}^4}\left(\mu_{20}\mu_{02} - \mu_{11}^2\right)$$
$$I_2 = \frac{1}{\mu_{00}^{10}}\left(\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{03}\mu_{21}^3 - 3\mu_{21}^2\mu_{12}^2\right)$$

- Region **corners** are often considered to be important features
- This is related to the **curvature** of the region boundary
- To consider curvature formally, let's go back to the continuous domain



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3 approaches to representation of 2D curves

(continuous domain)

- Implicit form $f(x, y) = 0$
- Explicit form $y = g(x)$
- Parametric form $\overline{\sigma}(s) = \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}, \quad a \leq s \leq b$

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- A <u>tangent vector</u> to the curve at point $\overline{\sigma}(s) = [x(s),\ y(s)]^T$ is

$$\frac{d\overline{\sigma}}{ds} = \left[\frac{dx}{ds},\ \frac{dy}{ds}\right]^T$$

(This is a unit vector when *s* represents distance along the curve)

- The <u>arc length</u> is $L = \int_a^b \left\| \frac{d\overline{\sigma}}{ds} \right\| ds = \int_a^b \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2}\, ds$

- A <u>normal vector</u> to the curve at point $\overline{\sigma}(s) = [x(s),\ y(s)]^T$ is

$$\frac{d^2\overline{\sigma}}{ds^2} = \left[\frac{d^2x}{ds^2},\ \frac{d^2y}{ds^2}\right]^T$$

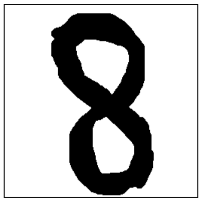
- The <u>curvature</u> is *K:*

$$K = \left\| \frac{d^2\overline{\sigma}}{ds^2} \right\| = \sqrt{\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2}$$

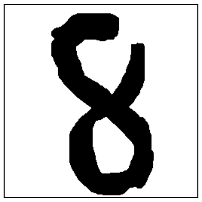
- The <u>radius of curvature</u> is $\frac{1}{K}$

- The previous slides described geometric properties
- There are also topological properties that do not depend on shape;   for example . . .

- **Euler number**
  = (number of connected components) - (number of holes)



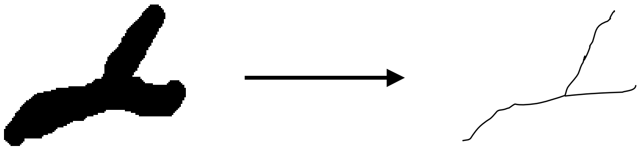
Euler number = -1



Euler number = 0

Region thinning

- Often it is desirable to reduce a region to a „skeleton“ of the original shape
- A **thinning operation** is used for this purpose
- Usually, this is useful only for elongated shapes



Common requirements:

- The skeleton of a connected component should also be a single connected set (i.e., a thinning procedure should not change the number of regions)
- A skeleton should be minimally connected (i.e., digital curves that are only 1 pixel thick)
- Pixels of the skeleton should lie near the center of the cross-section of the original region
- A thinning algorithm should not remove the end-points the skeleton
- Short branches ("spurs") should be avoided

- A well-known technique, the Medial Axis Transform [MAT], is very susceptible to noise
- The Zhang-Suen method (1984) has been used in practical applications

Summary

- A **region** is a connected portion of an image
- Regions are often obtained using a thresholding operation
- After thresholding, **region labeling** needs to be performed so that pixels of one region can be distinguished from pixels of other regions
- Many features can be computed for a region, and these can be used in object recognition
- In some cases, <u>thinning</u> is used to obtain a skeleton of a region
- **Run-length codes** can be used to represent a binary image
- **Chain codes** can be used to represent region boundaries