Problem 1: State and prove Lyapunov's theorem for autonomous systems. (10 points)

Problem 2: State Lasalle's theorem (5 points)

Problem 3: State and prove Lyapunov's theorem for nonautonomous systems. You don't have to prove the asymptotic part. (10 points)

Problem 4:

(a) Consider

$$\dot{x}_1 = -x_1 + x_2 d(t)$$
$$\dot{x}_2 = -x_1 d(t)$$

Prove that $Lt_{t\to\infty} x_1(t) = 0$ (10 points)

(b) Consider

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -h_1(x_1) - h_2(x_2)$$

$$h_i(0) = 0, \quad y h_i(y) > 0, \quad \forall y \neq 0, y \in (-a, a)$$

Use $V(x) = \int_0^{x_1} h_1(y) dy + \frac{1}{2} x_2^2$ and show that the origin is asymptotically stable. (10 points)