

# ECE 6414, Nonlinear and Adaptive Control Systems

## Fall 2003, 3 credits Final Exam

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1. Consider the system

$$\dot{x}_1 = \sin x_2 + \sqrt{t+1}x_2$$

$$\dot{x}_2 = \alpha_1(t) + x_1^4 \cos x_2 + \alpha_2 u$$

Design the control law  $u$  in order to track the desired function for state  $x_1$  given by  $x_{d1}$ . The functions  $\alpha_1(t)$  and  $\alpha_2(t)$  are unknown but bounded by  $|\alpha_1(t)| \leq 10$  and  $1 \leq \alpha_2(t) \leq 2$ . (10 points)

2. Problem 14.31
3. Problem 14.37
4. Consider the system

$$\dot{x} = f(x) + a + bu$$

The function  $f(x)$  is unknown and is estimated by  $\hat{f}(x)$  such that  $|f(x) - \hat{f}(x)| \leq F(x)$ . The parameter  $a$  is unknown but is either a constant or slowly time varying. The parameter  $b$  is known. Define  $\tilde{x} = x - x_d$ , where subscript  $d$  indicated the desired variable, and  $\tilde{a} = a - a_*$ , where the subscript  $*$  indicates the actual value. Choose the sliding variable as  $s = \tilde{x}$  and a candidate Lyapunov function as  $v = \frac{s^2}{2} + k \frac{\tilde{a}^2}{2}$ . Design an adaptive control law for the system. Show that  $\lim_{t \rightarrow \infty} \tilde{x} \rightarrow 0$ . (Show any extra assumptions that were needed to come to this conclusion). (10 points)