ECE 6444: Hybrid Control Systems
Spring 2003, 3 credits, CRN: 16271
HW#2
Due Date: Feb 27th, 2003

Dr. Pushkin Kachroo
The Bradley Department of Electrical and Computer Engineering, Virginia Tech,
Blacksburg, VA 24061-0111, pushkin@vt.edu

Problem#1: Prove that guarantee property on a non-empty set is the complement of the safety property of the complement of the same non-empty set. (Use Section 6.1 of the textbook as a reference and also to see a temporal logic equation of the same statement). (10 points)

Problem#2: Write the following in their expanded forms.
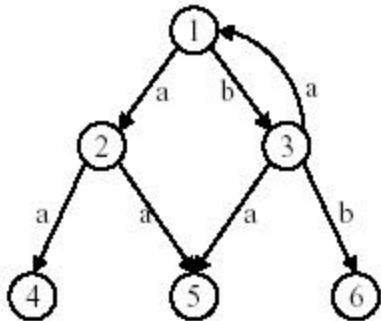
$$\Box \{ x_1 \geq 0 \}$$
$$\Diamond \{ x_1 = 0 \}$$
$$\Box \Diamond p \{ x_1 = 0 \}$$
$$\Diamond \Box p \{ x_1 < 1 \}$$

(10 points)

Problem#3: (a) Show all the steps for reachability algorithm applied to state 2 for the following system. (5 points)


(b) Show all the steps for the backward reachability algorithm applied to state 2 for the same system. (5 points)

(c) Show all the steps for the invariant set algorithm applied to set {1,3,4,5,6} for the same system. (5 points)

(d) What is the relationship between the result from part (b) and (c)? (5 points)

Problem#4: Apply the reachability algorithm to the tank system where the initial state is given below.
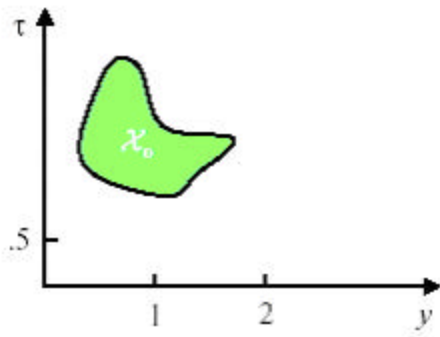
(10 points)
Problem#5: Using the Lyapunov function
$$V(x_1, x_2) = x_1^2 + x_2^2$$
study the stability of the following system.
$$\dot{x}_1 = x_1(x_1^2 + x_2^2 - 1) - x_2$$
$$\dot{x}_2 = x_1 + x_2(x_1^2 + x_2^2 - 1)$$
(10 points)