

Problem 1 (10 Points) What is the probability that two playing cards picked at random from a full deck includes an ace and a Queen?

Problem 2 (10 Points) Is the following true? If it is, provide the proof of the theorem, otherwise, provide a counter-example.

Theorem: If A_1, A_2, \dots is an “increasing sequence” of events, i.e., a sequence such that $A_1 \subset A_2 \subset \dots$, then

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{n \rightarrow \infty} P(A_n)$$

Problem 3 (10 Points) State and prove the first Borel-Cantelli lemma.