

Problem 1 (10 Points) (a) Find the steady state probabilities p_1^* and p_2^* for a Markov chain with $p_{11} = 0.25$ and $p_{21} = 0.5$.

(b) Give a one (or two) sentence reasoning for the following statement: "All states of a finite Markov chain which has all states accessible from each other, are all persistent."

Problem 2 (10 Points) For a continuous Markov process, prove that $P\{\tau > t\} = e^{-\lambda t}$, $t \geq 0$, where λ is a non-negative constant, and τ is the time it takes the process to leave a given state.

Problem 3 (10 Points) (a) Given a system that has two states A and B such that the probability of being in state A is x . For what value(s) of x does the system have the least information, and for what value(s) of x does the system have the most information. In other words, the question is that what probability distributions have the least information and which one has the most.

(b) Given a mixed two person zero-sum game with a payoff matrix v_{ij} with each player having two choices, find the optimal probability for choice-1 for player-1.