E8.15 Find $\mathbf{V}_o$ in Fig. E8.15.

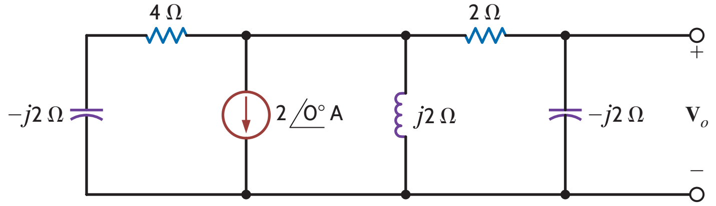


FIGURE E8.15

Answer:

$\mathbf{V}_o = 2.98\angle -153.43°$ V.

E8.16 In the network in Fig. E8.16, $\mathbf{V}_o$ is known to be $8\angle 45°$ V. Compute $\mathbf{V}_S$.

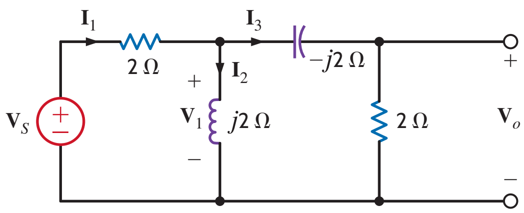


FIGURE E8.16

Answer:

$\mathbf{V}_S = 17.89\angle -18.43°$ V.

8.8 Analysis Techniques

In this section we revisit the circuit analysis methods that were successfully applied earlier to dc circuits and illustrate their applicability to ac steady-state analysis. We will present these techniques through examples in which all the theorems, together with nodal analysis and loop analysis, are used to obtain a solution.

EXAMPLE 8.15

Let us determine the current $\mathbf{I}_o$ in the network in **Fig. 8.17a** using nodal analysis, loop analysis, superposition, source exchange, Thévenin's theorem, and Norton's theorem.

Solution

1. ***Nodal Analysis*** We begin with a nodal analysis of the network [see HINT 8.21]. The KCL equation for the supernode that includes the voltage source is

$$\frac{\mathbf{V}_1}{1+j} - 2\angle 0° + \frac{\mathbf{V}_2}{1} + \frac{\mathbf{V}_2}{1-j} = 0$$

and the associated KVL constraint equation is

$$\mathbf{V}_1 + 6\angle 0° = \mathbf{V}_2$$

HINT 8.21

Summing the current, leaving the supernode. Outbound currents have a positive sign.

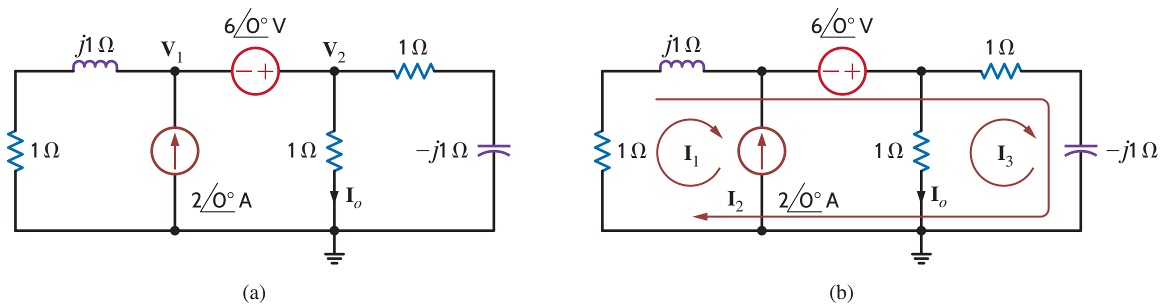


FIGURE 8.17 Circuits used in Example 8.15 for node and loop analysis.

The two equations in matrix form are

$$\begin{pmatrix} 0.5-0.5j & 1.5+0.5j \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

The MATLAB solution is then

```
>> Y = [0.5-0.5j  1.5+0.5j;  1  -1]
   Y =
        0.5000-0.5000i          1.5000+0.5000i
        1.0000                 -1.0000
>> I = [2; -6]
   I =
         2
        -6
>> V = inv(Y)*I
   V =
        -3.5000-1.5000i
         2.5000-1.5000i
>> abs  (V)
         3.8079
         2.9155
>> 180*phase(v)/pi
   Ans  =
        -156.8014
         -30.9638
```

And since $\mathbf{I}_0 = \mathbf{V}_2/1$, $\mathbf{I}_0 = 2.9155\,\underline{/-30.9638°}$ A

```
>> 180*phase(V)/pi

   ans =
        -156.8014
         -30.9638
```

And since $\mathbf{I}_0 = \mathbf{V}_2/1$, $\mathbf{I}_0 = 2.9155\,\underline{/-30.9638°}$ A.

HINT 8.22

Just as in a dc analysis, the loop equations assume that a decrease in potential level is + and an increase is −.

2. ***Loop Analysis*** The network in **Fig. 8.17b** is used to perform a loop analysis [see **HINT 8.22**]. Note that one loop current is selected that passes through the independent current source. The three loop equations are

$$\mathbf{I}_1 = -2\,\underline{/0°}$$

$$1(\mathbf{I}_1+\mathbf{I}_2) + j1(\mathbf{I}_1+\mathbf{I}_2) - 6\,\underline{/0°} + 1(\mathbf{I}_2+\mathbf{I}_3) - j1(\mathbf{I}_2+\mathbf{I}_3) = 0$$

$$1\mathbf{I}_3 + 1(\mathbf{I}_2+\mathbf{I}_3) - j1(\mathbf{I}_2+\mathbf{I}_3) = 0$$

Combining the first two equations yields

$$\mathbf{I}_2(2) + \mathbf{I}_3(1-j) = 8 + 2j$$

The third loop equation can be simplified to the form

$$\mathbf{I}_2(1-j) + \mathbf{I}_3(2-j) = 0$$

The equations in matrix form are

$$\begin{bmatrix} 2 & 1-j \\ 1-j & 2-j \end{bmatrix}\begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}\;\begin{bmatrix} 8+2j \\ 0 \end{bmatrix}$$

The MATLAB solution is then

```
>> Z = [2 1-j;  1-j  2-j]
   Z =
        2.0000                  1.0000 - 1.0000i
        1.0000 - 1.0000i        2.0000 - 1.0000i
```

```
>> V = [8+2*j; 0]
   V =
          8.0000  +  2.0000i
               0
>> I = inv(Z)*V
   I =
           4.5000  -  1.0000i
          -2.5000  +  1.5000i
>> abs (I)
   ans =
           4.6098
           2.9155
>> 180*phase(I)/pi
   ans =
             -12.5288
             149.0362
```

Therefore, $\mathbf{I}_3 = 2.9155\,\underline{/149.0362°}$ and $\mathbf{I}_0 = -\mathbf{I}_3 = 2.9155\,\underline{/-30.9638°}$ A.

3. ***Superposition*** In using superposition, we apply one independent source at a time [see **HINT 8.23**]. The network in which the current source acts alone is shown in **Fig. 8.18a**. By combining the two parallel impedances on each end of the network, we obtain the circuit in **Fig. 8.18b**, where

$$\mathbf{Z}' = \frac{(1+j)(1-j)}{(1+j)+(1-j)} = 1\ \Omega$$

Therefore, using current division,

$$\mathbf{I}'_0 = 1\ \underline{/0°}\ \text{A}$$

The circuit in which the voltage source acts alone is shown in **Fig. 8.18c**. The voltage $\mathbf{V}''_1$ obtained using voltage division is

$$\mathbf{V}''_1 = \frac{(6\underline{/0°})\left[\dfrac{1(1-j)}{1+1-j}\right]}{1+j+\left[\dfrac{1(1-j)}{1+1-j}\right]} = \frac{6(1-j)}{4}\ \text{V}$$

and hence,

$$\mathbf{I}''_0 = \frac{6}{4}(1-j)\ \text{A}$$

HINT 8.23

In applying superposition in this case, each source is applied independently, and the results are added to obtain the solution.

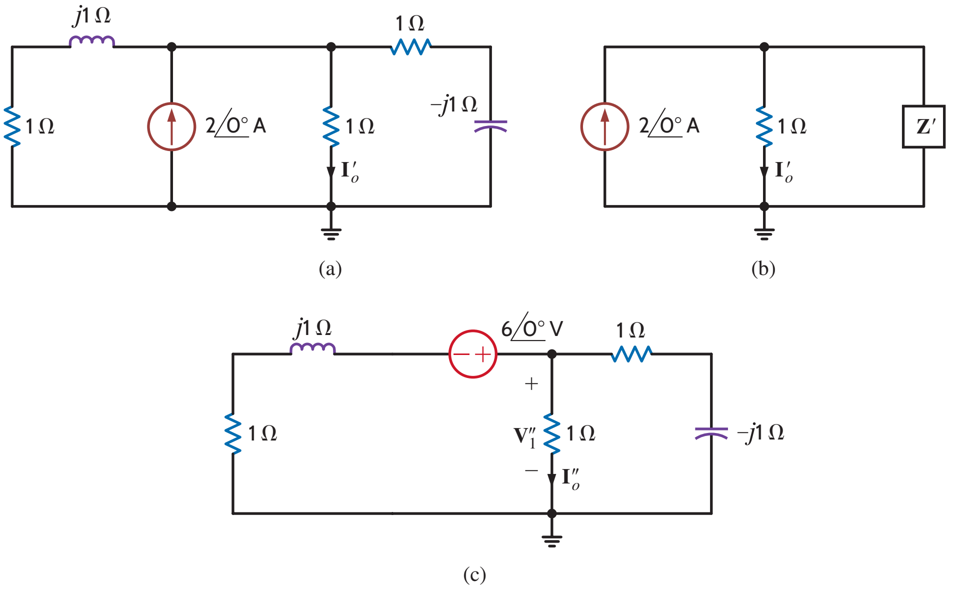


FIGURE 8.18 Circuits used in Example 8.15 for a superposition analysis.

HINT 8.24

In source exchange, a voltage source in series with an impedance can be exchanged for a current source in parallel with the impedance and vice versa. Repeated application systematically reduces the number of circuit elements.

Then

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = 1 + \frac{6}{4}(1 - j) = 2.9155\underline{/-30.9638°}\text{ A.}$$

4. ***Source Exchange*** As a first step in the source exchange approach, we exchange the current source and parallel impedance for a voltage source in series with the impedance, as shown in **Fig. 8.19a** [see **HINT 8.24**].

Adding the two voltage sources and transforming them and the series impedance into a current source in parallel with that impedance are shown in **Fig. 8.19b**. Combining the two impedances that are in parallel with the 1-Ω resistor produces the network in **Fig. 8.19c**, where

$$\mathbf{Z} = \frac{(1 + j)(1 - j)}{1 + j + 1 - j} = 1\,\Omega$$

Therefore, using current division,

$$\mathbf{I}_o = \left(\frac{8 + 2j}{1 + j}\right)\left(\frac{1}{2}\right) = \frac{4 + j}{1 + j}$$
$$= 2.9155\underline{/-30.9638°}\text{ A}$$

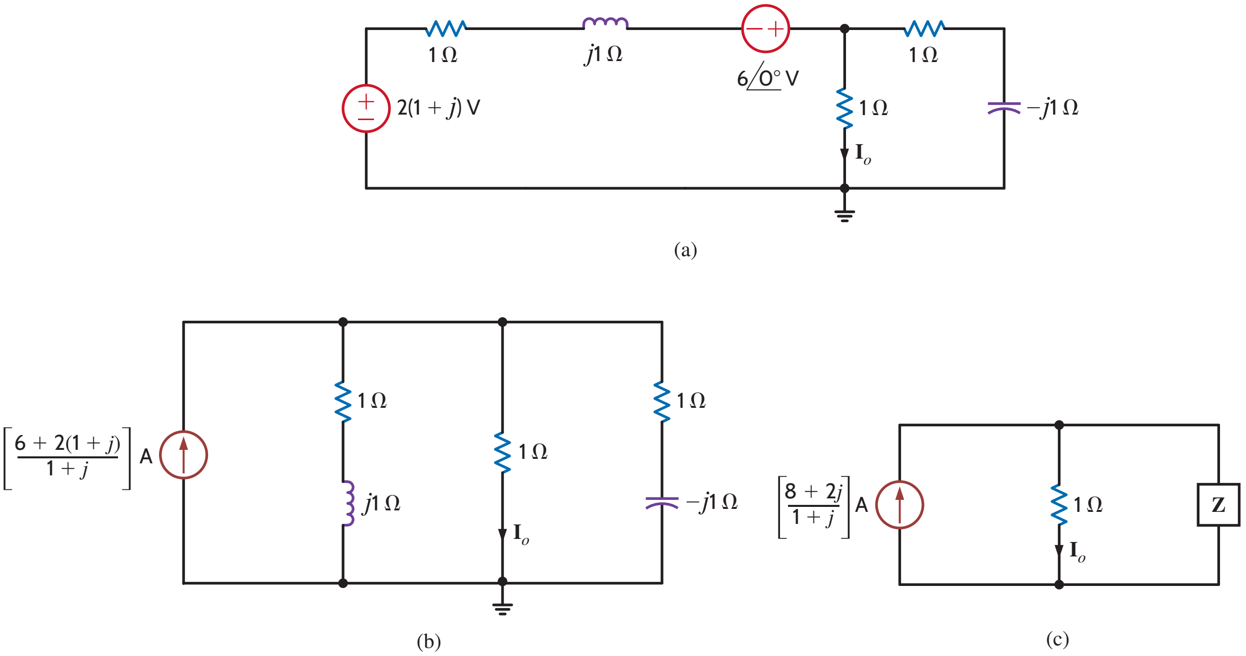
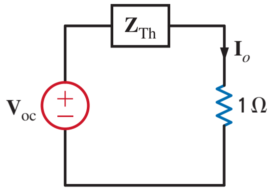


FIGURE 8.19 Circuits used in Example 8.15 for a source exchange analysis.

HINT 8.25

In this Thévenin analysis,

1. **Remove the 1-Ω load and find the voltage across the open terminals, $\mathbf{V}_{oc}$.**
2. **Determine the impedance $\mathbf{Z}_{Th}$ at the open terminals with all sources made zero.**
3. **Construct the following circuit and determine $\mathbf{I}_o$.**



5. ***Thévenin Analysis*** In applying Thévenin's theorem to the circuit in Fig. 8.17a, we first find the open-circuit voltage, $\mathbf{V}_{oc}$, as shown in **Fig. 8.20a** [see **HINT 8.25**]. To simplify the analysis, we perform a source exchange on the left end of the network, which results in the circuit in **Fig. 8.20b**. Now using voltage division,

$$\mathbf{V}_{oc} = [6 + 2(1 + j)]\left[\frac{1 - j}{1 - j + 1 + j}\right]$$

or

$$\mathbf{V}_{oc} = (5 - 3j)\text{ V}$$

The Thévenin equivalent impedance, $\mathbf{Z}_{Th}$, obtained at the open-circuit terminals when the current source is replaced with an open circuit and the voltage source is replaced with a short circuit, is shown in **Fig. 8.20c** and calculated to be

$$\mathbf{Z}_{Th} = \frac{(1 + j)(1 - j)}{1 + j + 1 - j} = 1\,\Omega$$

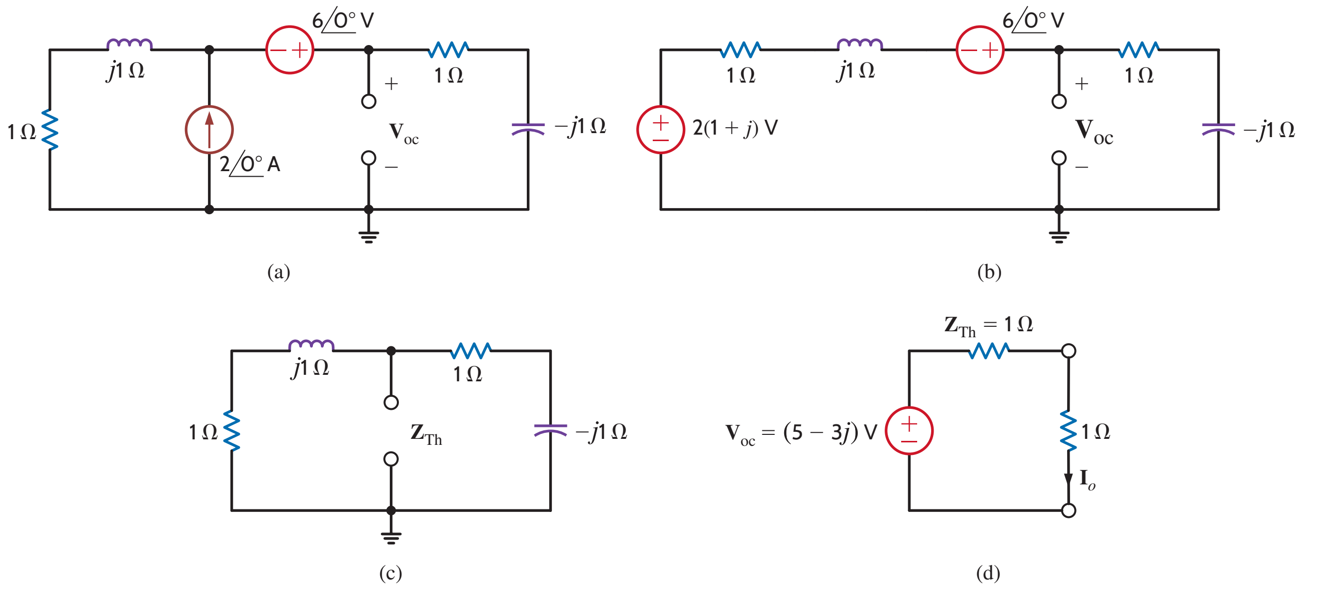


FIGURE 8.20 Circuits used in Example 8.15 for a Thévenin analysis.

Connecting the Thévenin equivalent circuit to the 1-Ω resistor containing $\mathbf{I}_o$ in the original network yields the circuit in **Fig. 8.20d**. The current $\mathbf{I}_o$ is then

$$\mathbf{I}_o = 2.9155\angle{-30.9638°}\text{ A}$$

6. ***Norton Analysis*** Finally, in applying Norton's theorem to the circuit in Fig. 8.17a, we calculate the short-circuit current, $\mathbf{I}_{sc}$, using the network in **Fig. 8.21a** [see **HINT 8.26**]. Note that because of the short circuit, the voltage source is directly across the impedance in the left-most branch. Therefore,

$$\mathbf{I}_1 = \frac{6\angle 0°}{1 + j}$$

Then, using KCL,

$$\mathbf{I}_{sc} = \mathbf{I}_1 + 2\angle 0° = 2 + \frac{6}{1+j}$$

$$= \left(\frac{8 + 2j}{1 + j}\right)\text{ A}$$

The Thévenin equivalent impedance, $\mathbf{Z}_{Th}$, is known to be 1 Ω and, therefore, connecting the Norton equivalent to the 1-Ω resistor containing $\mathbf{I}_o$ yields the network in **Fig. 8.21b**. Using current division, we find that

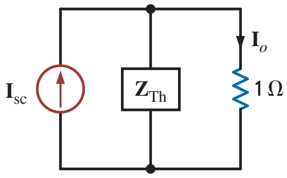
$$\mathbf{I}_o = \frac{1}{2}\left(\frac{8 + 2j}{1 + j}\right)$$

$$= 2.9155\angle{-30.9638°}\text{ A}$$

HINT 8.26

In this Norton analysis,

1. **Remove the 1-Ω load and find the current $\mathbf{I}_{sc}$ through the short-circuited terminals.**
2. **Determine the impedance $\mathbf{Z}_{Th}$ at the open load terminals with all sources made zero.**
3. **Construct the following circuit and determine $\mathbf{I}_o$.**



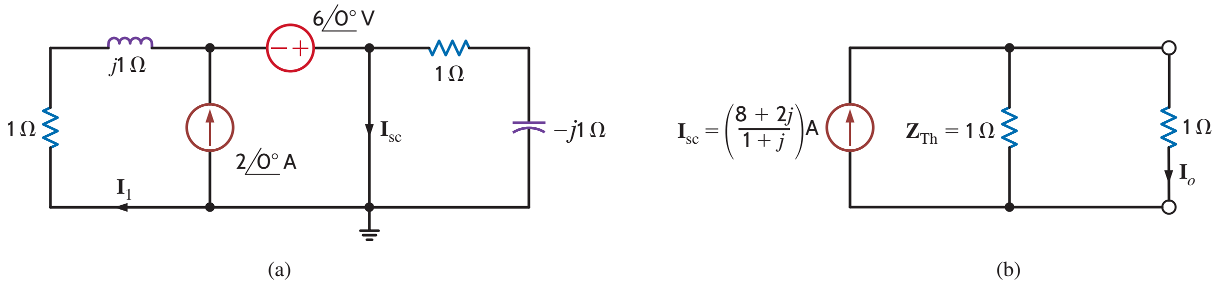


FIGURE 8.21 Circuits used in Example 8.15 for a Norton analysis.