in which **G** represents the coefficient matrix of conductances, **V** is the column vector of node voltages, and **I** is the column vector of currents on the right-hand side.

Here again, we could write the equations directly in standard or matrix form using the short cut method because the circuit contains only resistances and independent current sources.

The MATLAB solution is:

```
>> clear
>> G = [0.35 -0.2 -0.05; -0.2 0.3 -0.1; -0.05 -0.1 0.35];
>> % A semicolon at the end of a command suppresses the
>> % MATLAB response.
>> I = [0; 10; 0];
>> V = G\I
V =
    45.4545
    72.7273
    27.2727
>> % Finally, we calculate the current.
>> Ix = (V(1) - V(3))/20
Ix =
     0.9091
```

Alternatively, you can use the same commands with LabVIEW MathScript to obtain the answers. ■

Exercise 2.9 Repeat the analysis of the circuit of Example 2.8, using the reference node and node voltages shown in Figure 2.22. **a.** First write the network equations. **b.** Put the network equations into standard form. **c.** Solve for v_1, v_2, and v_3. (The values will be different than those we found in Example 2.8 because v_1, v_2, and v_3 are not the same voltages in the two figures.) **d.** Find i_x. (Of course, i_x is the same in both figures, so it should have the same value.)

Answer

a.

$$\frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0$$
$$\frac{v_2 - v_1}{10} + 10 + \frac{v_2 - v_3}{5} = 0$$
$$\frac{v_3 - v_1}{20} + \frac{v_3}{10} + \frac{v_3 - v_2}{5} = 0$$

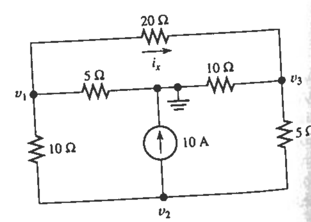

Figure 2.22 Circuit of Example 2.8 with a different choice for the reference node. See Exercise 2.9.

b.

$$\begin{aligned} 0.35v_1 - 0.10v_2 - 0.05v_3 &= 0 \\ -0.10v_1 + 0.30v_2 - 0.20v_3 &= -10 \\ -0.05v_1 - 0.20v_2 + 0.35v_3 &= 0 \end{aligned}$$

c. $v_1 = -27.27$; $v_2 = -72.73$; $v_3 = -45.45$

d. $i_x = 0.909$ A

□

Circuits with Voltage Sources

When a circuit contains a single voltage source, we can often pick the reference node at one end of the source, and then we have one less unknown node voltage for which to solve.

Example 2.9 Node-Voltage Analysis

Write the equations for the network shown in Figure 2.23 and put them into standard form.

Solution Notice that we have selected the reference node at the bottom end of the voltage source. Thus, the voltage at node 3 is known to be 10 V, and we do not need to assign a variable for that node.

Writing current equations at nodes 1 and 2, we obtain

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1$$
$$\frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0$$

Now if we group terms and place the constants on the right-hand sides of the equations, we have

$$0.7v_1 - 0.2v_2 = 6$$
$$-0.2v_1 + 0.5v_2 = 1$$

Thus, we have obtained the equations needed to solve for v_1 and v_2 in standard form. ■

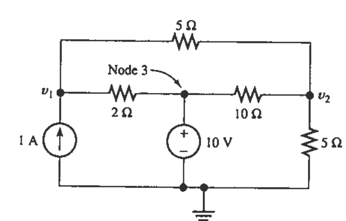

Figure 2.23 Circuit for Example 2.9.

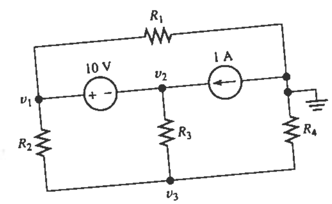

Figure 2.27 Circuit for Exercise 2.13.

KCL for the supernode enclosing the 10-V source:

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

KCL for node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

KCL at the reference node:

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

For independence, the set must include the KVL equation. Any two of the three KCL equations can be used to complete the three-equation set. (The three KCL equations use all of the network nodes and, therefore, do not form an independent set.) □

Circuits with Controlled Sources

Controlled sources present a slight additional complication of the node-voltage technique. (Recall that the value of a controlled source depends on a current or voltage elsewhere in the network.) In applying node-voltage analysis, first we write equations exactly as we have done for networks with independent sources. Then, we express the controlling variable in terms of the node-voltage variables and substitute into the network equations. We illustrate with two examples.

Example 2.10 Node-Voltage Analysis with a Dependent Source

Write an independent set of equations for the node voltages shown in Figure 2.28.

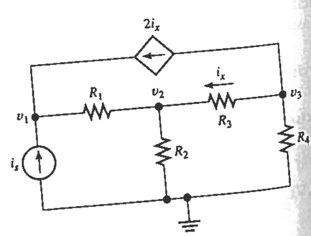

Figure 2.28 Circuit containing a current-controlled current source. See Example 2.10.

Solution First, we write KCL equations at each node, including the current of the controlled source just as if it were an ordinary current source:

$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x \quad (2.39)$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0 \quad (2.40)$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0 \quad (2.41)$$

Next, we find an expression for the controlling variable i_x in terms of the node voltages. Notice that i_x is the current flowing away from node 3 through R_3. Thus, we can write

$$i_x = \frac{v_3 - v_2}{R_3} \quad (2.42)$$

Finally, we use Equation 2.42 to substitute into Equations 2.39, 2.40, and 2.41. Thus, we obtain the required equation set:

$$\frac{v_1 - v_2}{R_1} = i_s + 2\frac{v_3 - v_2}{R_3} \quad (2.43)$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0 \quad (2.44)$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2\frac{v_3 - v_2}{R_3} = 0 \quad (2.45)$$

Assuming that the value of i_s and the resistances are known, we could put this set of equations into standard form and solve for v_1, v_2, and v_3. ■

Example 2.11 Node-Voltage Analysis with a Dependent Source

Write an independent set of equations for the node voltages shown in Figure 2.29.

Solution First, we ignore the fact that the voltage source is a dependent source and write equations just as we would for a circuit with independent sources. We cannot write a current equation at either node 1 or node 2, because of the voltage source connected between them. However, we can write a KVL equation:

$$-v_1 + 0.5v_x + v_2 = 0 \quad (2.46)$$

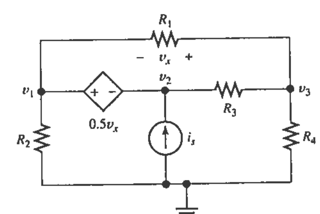

Figure 2.29 Circuit containing a voltage-controlled voltage source. See Example 2.11.

Then, we use KCL to write current equations. For a supernode enclosing the controlled voltage source,

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

For node 3,

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0 \qquad (2.47)$$

For the reference node,

$$\frac{v_1}{R_2} + \frac{v_3}{R_4} = i_s \qquad (2.48)$$

Of course, these current equations are dependent because we have used all four nodes in writing them. We must use Equation 2.46 and two of the KCL equations to form an independent set. However, Equation 2.46 contains the controlling variable v_x, which must be eliminated before we have equations in terms of the node voltages.

Thus, our next step is to write an expression for the controlling variable v_x in terms of the node voltages. Notice that v_1, v_x, and v_3 form a closed loop. Traveling clockwise and summing voltages, we have

$$-v_1 - v_x + v_3 = 0$$

Solving for v_x, we obtain

$$v_x = v_3 - v_1$$

Now if we substitute into Equation 2.46, we get

$$v_1 = 0.5(v_3 - v_1) + v_2 \qquad (2.49)$$

Equation 2.49 along with any two of the KCL equations forms an independent set that can be solved for the node voltages. ■

Using the principles we have discussed in this section, we can write node-voltage equations for any network consisting of sources and resistances. Thus, given a computer or calculator to help in solving the equations, we can compute the currents and voltages for any network.

Next, we summarize the steps in analyzing circuits by the node-voltage technique:

Here is a convenient step-by-step guide to node-voltage analysis.

1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.
2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.
4. Put the equations into standard form and solve for the node voltages.

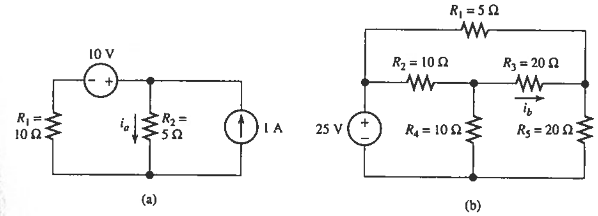

Figure 2.30 Circuits for Exercise 2.14.

5. Use the values found for the node voltages to calculate any other currents or voltages of interest.

Exercise 2.14 Use the node-voltage technique to solve for the currents labeled in the circuits shown in Figure 2.30.
Answer **a.** $i_a = 1.33$ A; **b.** $i_b = -0.259$ A. □

Exercise 2.15 Use the node-voltage technique to solve for the values of i_x and i_y in Figure 2.31.
Answer $i_x = 0.5$ A, $i_y = 2.31$ A. □

Using the MATLAB Symbolic Toolbox to Obtain Symbolic Solutions

If the Symbolic Toolbox is included with your version of MATLAB, you can use it to solve node voltage and other equations symbolically. (LabVIEW MathScript does not have symbolic mathematics capabilities.) We illustrate by solving Equations 2.43, 2.44, and 2.45 from Example 2.10 on page 74.

For help with a command such as "solve" simply type "help solve" at the command prompt.

```
>> clear
>> % First we clear the workspace, then we enter the equations into
>> % the solve command followed by the variables for which we want
>> % to solve.
>> [V1, V2, V3] = solve('(V1 - V2)/R1 = Is + 2*(V3 - V2)/R3', ...
           '(V2 - V1)/R1 + V2/R2 + (V2 - V3)/R3 = 0', ...
           '(V3 - V2)/R3 + V3/R4 + 2*(V3 - V2)/R3 = 0',...
           'V1','V2','V3')
```

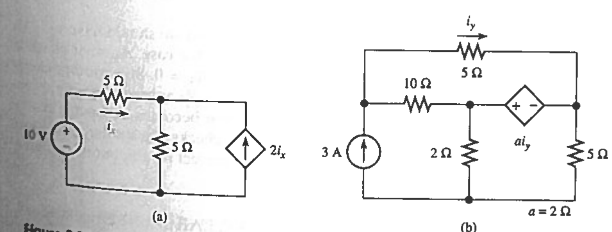

Figure 2.31 Circuits for Exercise 2.15.

Applying KCL to the node at the top end of R_3, we have

$$i_1 = i_2 + i_3 \quad (2.52)$$

Next, we solve Equation 2.52 for i_3 and substitute into Equations 2.50 and 2.51. This yields the following two equations:

$$R_1 i_1 + R_3(i_1 - i_2) = v_A \quad (2.53)$$

$$-R_3(i_1 - i_2) + R_2 i_2 = -v_B \quad (2.54)$$

Thus, we have used the KCL equation to reduce the KVL equations to two equations in two unknowns.

When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.

Now, consider the mesh currents i_1 and i_2 shown in Figure 2.32(b). As indicated in the figure, mesh currents are considered to flow around closed paths. Hence, mesh currents automatically satisfy KCL. *When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.* Thus, assuming a reference direction pointing downward, the current in R_3 is $(i_1 - i_2)$. Thus, $v_3 = R_3(i_1 - i_2)$. Now if we follow i_1 around its loop and apply KVL, we get Equation 2.53 directly. Similarly, following i_2, we obtain Equation 2.54 directly.

Because mesh currents automatically satisfy KCL, some work is saved in writing and solving the network equations. The circuit of Figure 2.32 is fairly simple, and the advantage of mesh currents is not great. However, for more complex networks, the advantage can be quite significant.

Choosing the Mesh Currents

We usually choose the current variables to flow clockwise around the periphery of each of the open areas of the circuit diagram.

For a planar circuit, we can choose the current variables to flow through the elements around the periphery of each of the open areas of the circuit diagram. For consistency, we usually define the mesh currents to flow clockwise.

Two networks and suitable choices for the mesh currents are shown in Figure 2.33. When a network is drawn with no crossing elements, it resembles a window, with each open area corresponding to a pane of glass. Sometimes it is said that the mesh currents are defined by "soaping the window panes."

Keep in mind that, if two mesh currents flow through a circuit element, we consider the current in that element to be the algebraic sum of the mesh currents. For example, in Figure 2.33(a), the current in R_2 referenced to the left is $i_3 - i_1$. Furthermore, the current referenced upward in R_3 is $i_2 - i_1$.

Exercise 2.17 Consider the circuit shown in Figure 2.33(b). In terms of the mesh currents, find the current in **a.** R_2 referenced upward; **b.** R_4 referenced to the right; **c.** R_8 referenced downward; **d.** R_8 referenced upward.
Answer **a.** $i_4 - i_1$; **b.** $i_2 - i_1$; **c.** $i_3 - i_4$; **d.** $i_4 - i_3$. [Notice that the answer for part (d) is the negative of the answer for part (c).]

Writing Equations to Solve for Mesh Currents

If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL. (We do not need to apply KCL because the mesh currents flow out of each node that they flow into.)

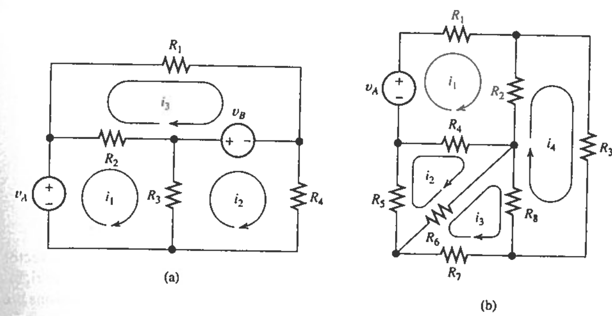

Figure 2.33 Two circuits and their mesh-current variables.

Example 2.12 Mesh-Current Analysis

Write the equations needed to solve for the mesh currents in Figure 2.33(a).

If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL.

Solution Using a pattern in solving networks by the mesh-current method helps to avoid errors. Part of the pattern that we use is to select the mesh currents to flow clockwise. Then, we write a KVL equation for each mesh, going around the meshes clockwise. As usual, we add a voltage if its positive reference is encountered first in traveling around the mesh, and we subtract the voltage if the negative reference is encountered first. Our pattern is always to take the first end of each resistor encountered as the positive reference for its voltage. Thus, we are always adding the resistor voltages.

For example, in mesh 1 of Figure 2.33(a), we first encounter the left-hand end of R_2. The voltage across R_2 referenced positive on its left-hand end is $R_2(i_1 - i_3)$. Similarly, we encounter the top end of R_3 first, and the voltage across R_3 referenced positive at the top end is $R_3(i_1 - i_2)$. By using this pattern, we add a term for each resistor in the KVL equation, consisting of the resistance times the current in the mesh under consideration minus the current in the adjacent mesh (if any). Using this pattern for mesh 1 of Figure 2.33(a), we have

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

Similarly, for mesh 2, we obtain

$$R_3(i_2 - i_1) + R_4 i_2 + v_B = 0$$

Finally, for mesh 3, we have

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

Notice that we have taken the positive reference for the voltage across R_3 at the top in writing the equation for mesh 1 and at the bottom for mesh 3. This is not an error because the terms for R_3 in the two equations are opposite in sign.

In standard form, the equations become:

$$(R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 = v_A$$
$$-R_3 i_1 + (R_3 + R_4)i_2 = -v_B$$
$$-R_2 i_1 + (R_1 + R_2)i_3 = v_B$$

In matrix form, we have

$$\begin{bmatrix} (R_2+R_3) & -R_3 & -R_2 \\ -R_3 & (R_3+R_4) & 0 \\ -R_2 & 0 & (R_1+R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_A \\ -v_B \\ v_B \end{bmatrix}$$

Often, we use **R** to represent the coefficient matrix, **I** to represent the column vector of mesh currents, and **V** to represent the column vector of the terms on the right-hand sides of the equations in standard form. Then, the mesh-current equations are represented as:

$$\mathbf{RI} = \mathbf{V}$$

We refer to the element of the ith row and jth column of **R** as r_{ij}. ■

Exercise 2.18 Write the equations for the mesh currents in Figure 2.32(b) and put them into matrix form.

Answer Following each mesh current in turn, we obtain

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$
$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$
$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$
$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

$$\begin{bmatrix} (R_1+R_2+R_4) & -R_4 & 0 & -R_2 \\ -R_4 & (R_4+R_5+R_6) & -R_6 & 0 \\ 0 & -R_6 & (R_6+R_7+R_8) & -R_8 \\ -R_2 & 0 & -R_8 & (R_2+R_3+R_8) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} v_A \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.55)$$

□

Solving Mesh Equations

After we write the mesh-current equations, we can solve them by using the methods that we discussed in Section 2.4 for the node-voltage approach. We illustrate with a simple example.

Example 2.13 Mesh-Current Analysis

Solve for the current in each element of the circuit shown in Figure 2.34.

Solution First, we select the mesh currents. Following our standard pattern, we define the mesh currents to flow clockwise around each mesh of the circuit. Then, we write a KVL equation around mesh 1:

$$20(i_1 - i_3) + 10(i_1 - i_2) - 70 = 0 \quad (2.5$$

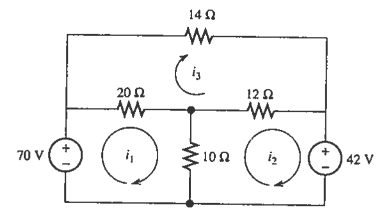

Figure 2.34 Circuit of Example 2.13.

For meshes 2 and 3, we have:

$$10(i_2 - i_1) + 12(i_2 - i_3) + 42 = 0 \quad (2.57)$$
$$20(i_3 - i_1) + 14i_3 + 12(i_3 - i_2) = 0 \quad (2.58)$$

Putting the equations into standard form, we have:

$$30i_1 - 10i_2 - 20i_3 = 70 \quad (2.59)$$
$$-10i_1 + 22i_2 - 12i_3 = -42 \quad (2.60)$$
$$-20i_1 - 12i_2 + 46i_3 = 0 \quad (2.61)$$

In matrix form, the equations become:

$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix}$$

These equations can be solved in a variety of ways. We will demonstrate using MATLAB. (The same results can be obtained by using these same commands in LabVIEW MathScript.) We use **R** for the coefficient matrix, because the coefficients often are resistances. Similarly, we use **V** for the column vector for the right-hand side of the equations and **I** for the column vector of the mesh currents. The commands and results are:

```
>> R = [30 -10 -20; -10 22 -12; -20 -12 46];
>> V = [70; -42; 0];
>> I = R\V % Try to avoid using i, which represents the square root of
>> % -1 in MATLAB.
I =
    4.0000
    1.0000
    2.0000
```

Thus, the values of the mesh currents are $i_1 = 4\,\text{A}$, $i_2 = 1\,\text{A}$, and $i_3 = 2\,\text{A}$. Next, we can find the current in any element. For example, the current flowing downward in the 10-Ω resistance is $i_1 - i_2 = 3\,\text{A}$. ■

Exercise 2.19 Use mesh currents to solve for the current flowing through the 10-Ω resistance in Figure 2.35. Check your answer by combining resistances in series and parallel to solve the circuit. Check a second time by using node voltages.