

Problem 1 (15 Points)

1. Derive the electromagnetic wave equation for the electric field in a charge and current free medium from Maxwell's equations. (5 points)
2. Obtain the Helmholtz's equation for the Electric field phasor from the wave equation. (5 points)
3. State the electric field phasor solution for the Helmholtz's equation for the forward moving wave in z-direction with the field component only in the x-direction and its corresponding magnetic field phasor. (5 points)

Problem 2 (20 Points)

1. Derive the expression for Poynting's theorem for power flow of an electromagnetic wave. (10 Points)
2. What is the direction of the propagation of a wave whose electric field is given by $\mathbf{E}(\mathbf{R}) = \mathbf{E}_0 e^{-j\mathbf{K}\cdot\mathbf{R}}$? How are the vectors \mathbf{K} and \mathbf{E}_0 related? Find the corresponding magnetic field $\mathbf{H}(\mathbf{R})$. (5 points)
3. What type of polarization (type, and positive or negative if applicable) is in the wave, the electric field phasor for which is $E(z) = \mathbf{a}_x E_0 e^{-jkz} + \mathbf{a}_y j E_0 e^{-jkz}$. (5 Points)

Problem 3 (20 Points)

1. Consider the case of normal incidence of an electromagnetic wave travelling from a lossless medium to a perfect conductor. Assume the incidence electric field phasor of $\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z}$, derive or state the incident magnetic field phasor $\mathbf{H}_i(z)$, the reflected phasors, and the total sum of phasors in both media. Write the expression for the waves in space and time coordinates. (10 Points)
2. Consider the case of oblique incidence of an electromagnetic wave at a plane dielectric boundary, where the angle of incidence is θ_i , the angle of reflection θ_r , and the angle of transmission θ_t . Derive the Snell's law of reflection and refraction using speed of propagation and geometry. (10 Points)