Problem 1 (10 Points) Prove by using Ampere's circuital law that there can be no TEM waves in a hollow waveguide.

Problem 2 (10 Points) The cutoff frequency for a rectangular hollow waveguide is

$$f_{c_{mn}} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Using the speed of light to be $3.0 \times 10^8 m/s$ find the cutoff frequency for TE_{10} for $a = 1m$.

Problem 3 (10 Points) Prove for a waveguide propagation that

$$\sin\theta = \frac{m\lambda/2}{a}$$

Show that

$$\lambda_c = \frac{2a}{m}$$

and show $\sin\theta$ in terms of f and f_c.

Problem 4 (100 Points)

1. Write down the Maxwell's equations in phasor form from which the waveguide field equations are derived.
2. Write down $\mathbf{E}_s$ and $\mathbf{H}_s$ in terms of their three vector components such as E_{xs} etc.
3. Write all six components as shown for one component here $E_{xs} = E_x e^{-j\beta z}$, where $E_x(x, y)$ is independent of z.
4. Substitute these six components in the following set of PDEs obtained from the Maxwell's equations so as to obtain 4 equations in terms of E_x,E_y, E_z, H_x,H_y, and H_z.

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs}; \quad \frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = -j\omega\mu H_{ys}$$
$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\epsilon E_{xs}; \quad \frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\epsilon E_{ys}$$

5. Write E_x, E_y, H_x, and H_y in terms of H_z and E_Z.
6. Now, for TM mode, take $H_z = 0$. Now take $E_{zs} = E_z e^{-j\beta z}$. Use this expression to show that the Helmoltz equation $\nabla^2 E_s + \beta_u^2 E_s = 0$, where $\beta_u = \omega\sqrt{\mu\epsilon}$ can be written as

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\beta_u^2 - \beta^2)E_z = 0$$

7. Take $E_z(x, y) = XY$ and by substituting in the previous PDE, show what differential equations do X and Y satisfy.
8. Show that sum of sines and cosines satisfy those differential equations for X and Y.
9. Apply boundary conditions $X = 0$ at $x = 0$ and $x = a$, and also, $Y = 0$ at $y = 0$ and $y = b$ and derive the final form for X and Y.