Problem 1 (10 Points)
(a) Prove that the output $y(t)$ of a continuous time LTI system to an input $x(t)$ is given by $y(t) = h(t) * x(t)$ where $h(t)$ is the impulse response of the continuous system.
(b)Prove that the output $y[n]$ of a discrete time LTI system to an input $x[n]$ is given by $y[n] = h[n] * x[n]$ where $h[n]$ is the impulse response of the discrete system.

Problem 2 (20 Points)
(a) Prove that $u'(t) = \delta(t)$, where $u(t)$ is the unit step function, and $\delta(t)$ is the Dirac delta impulse.
(b) Prove that $u[n] - u[n-1] = \delta[n]$, where $u[n]$ is the discrete unit step function, and $\delta[n]$ is the discrete impulse.
(c) Prove that $s'(t) = h(t)$, where $s(t)$ is the step response, and $h(t)$ is the impulse response of an LTI system.
(c) Prove that $s[n]-s[n-1] = h([n]$, where $s[n]$ is the discrete step response, and $h[n]$ is the impulse response of a discrete LTI system.

Problem 3 (10 Points)
(a) Prove that e^{st} is an eigenfunction for a continuous time LTI system. Find the eigenvalue in terms of the impulse response of the system.
(b) Prove that z^n is an eigenfunction for a discrete time LTI system. Find the eigenvalue in terms of the impulse response of the system.

Problem 4 (15 Points) Given a periodic continuous function $f(t)$ with time period T, which is written as:

$$f(t) = \sum_{i=0}^{i=\infty} \alpha_i \psi_i(t)$$

(a) Prove that $\{\psi_i(t)\}$ is an orthonormal set, where $\psi_i(t) = e^{ji\omega_0 t}/2T$, such that $\omega_0 = 2\pi/T$.
(b) What is the formula for α_i?
(c) What is the angle between $f_1(t) = t$, and $f_2(t) = t^2$, where the domain of the functions is $[0, 1]$.