Problem 1 (10 Points)

1. Prove that any set of outer measure zero is measurable.
2. Prove that there are disjoint sets of real numbers A and B for which $m^*(A \cup B) < m^*(A) + m^*(B)$.

Problem 2 (10 Points)

1. Prove that the Cantor-Lebesgue function is not absolutely continuous.
2. Prove the existence of a Lebesgue measurable set which is not Borel.

Problem 3 (10 Points) Show that if f is a bounded function on E that belongs to $L^{p_1}(E)$, then it belongs to $L^{p_2}(E)$ for $p_2 > p_1$.

Problem 4 (10 Points) Is the space L^∞ separable? Prove your answer.

Problem 5 (10 Points) Show that strong L^p convergence imples weak L^p convergence, and also that the (weak) limit of a weak convergent sequence is unique.

Problem 6 (10 Points) Let E be the set of finite measure and $\delta > 0$. Then show that E is the disjoint union of a finite collection of sets, each of which has measure less than δ.