

**Problem 1** (10 Points) For  $m(E) < \infty$  and  $1 \leq p_1 < p_2 \leq \infty$  show that  $L^{p_2}(E) \subset L^{p_1}(E)$ .

**Problem 2** (10 Points) Show that if  $f$  is a bounded function on  $E$  that belongs to  $L^{p_1}(E)$ , then it belongs to  $L^{p_2}(E)$  for  $p_2 > p_1$ .

**Problem 3** (10 Points) Is the space  $\ell^\infty$  separable? Prove your answer.

**Problem 4** (10 Points) Show that strong  $L^p$  convergence implies weak  $L^p$  convergence, and also that the (weak) limit of a weak convergent sequence is unique.

**Problem 5** (10 Points) State Helly's theorem and give a sketch of its proof.