Problem 1 (10 Points) If the measurable space (X, M) contains the topology T on X, show that every continuous function is measurable.

Problem 2 (10 Points) For a measure space (X, M, μ) state and prove the Chebychev's inequality.

Problem 3 (10 Points) For a measure space (X, M, μ) state and prove the Montone Convergence theorem.

Problem 4 (10 Points) For a finite measure space (X, M, μ) let $\{E_k\}_{k=1}^n$ be a collection of measurable sets, and $\{c_k\}_{k=1}^n$ a collection of real numbers. For $E \in M$, define

$$\nu(E) = \sum_{k=1}^{n} c_k \cdot \mu(E \cap E_k)$$

Show that ν is absolutely continuous with respect to μ and find its Radon-Nikodym derivative $\frac{d\mu}{d\nu}$.