Problem 1 (10 Points) Let (X, M, μ) be a measure space and $1 < p \leq \infty$. If $\{f_n\}$ is a bounded sequence of functions in $L^p(X, \mu)$, then prove that $\{f_n\}$ is uniformly integrable over X.

Problem 2 (10 Points) Let $S : L^p(X, \mu) \to \mathbf{R}$ be a bounded linear functional, show the steps to prove that for every simple function ϕ belonging to $L^p(X, \mu)$, $S(\phi) = \int_X f\phi d\mu$.

Problem 3 (10 Points) Prove that every Borel set in $\mathbf{R}^n$ is Lebesgue measurable, and use that to show that if E is a measurable subset of $\mathbf{R}^n$ and $f : E \to \mathbf{R}$ be continuous, then f is measurable with respect to the n-dimensional Lebesgue measure.