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## MAT 709: Complex Function Theory - I

Fall 2007, 3 credits: Test 1

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*PROBLEM 1* : (10 points) For which  $n$  is  $i$  an  $n$ th root of unity?

*PROBLEM 2* : (10 points) Sketch the image under the spherical projection of a line of longitude  $X = \sqrt{1 - Z^2} \cos \theta$ ,  $Y = \sqrt{1 - Z^2} \sin \theta$ , for  $\theta$  fixed and  $-1 \leq Z \leq 1$ .

*PROBLEM 3* : (10 points) Consider the transformation  $\ln z$ . Show the transformations of (a) circles centered at the origin, (b) rays emanating from the origin, and (c)  $z$ -plane.

*PROBLEM 4* : (10 points) Let  $u(x, y) = \alpha$  and  $v(x, y) = \beta$ , where  $u$  and  $v$  are the real and imaginary parts of an analytic function  $f(z)$  and  $\alpha$  and  $\beta$  are any constants, represent two families of curves. Prove that the families are orthogonal.