MAT 710: Complex Function Theory - II

Spring 2008, 3 credits: Test 2

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PROBLEM 1 : (10 points) The two-dimensional wave equation for a membrane in polar coordinates is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} \right)$$

$$\textit{Boundary conditions:} u(R,t) = 0 \ \forall t \geq 0$$

$$\textit{Initial Conditions:} u(r,0) = f(r) \ \text{ and } \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$

Obtain Bessel's equation using separation of variables.

PROBLEM 2 : (10 points) Apply Frobenius method and derive the indicial equation for the following Euler-Cauchy equation.

$$x^2 y'' + b_0 x y' + c_0 y = 0$$

PROBLEM 3 : (10 points) Given that $y_0, y_1, \cdots$ are eigenfunctions of a Sturm-Liouville problem, so that the eigenfunctions are orthogonal with respect to weight $p(x)$, derive the coefficients of the generalized Fourier series for a given function $f(x)$.

PROBLEM 4 : (10 points) Prove that a necessary and sufficient condition for $\prod_{k=1}^{\infty}(1+|w_k|)$ to converge is that $\sum |w_k|$ converges.

PROBLEM 5 : (10 points) For the gamma function, which for $\text{Re}\{z\} > 0$ is given by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$$

prove that $\Gamma(z+1) = z\Gamma(z)$.